Estimation of the injected structure-borne sound power using inversely measured contact forces

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Abstract
In previous work, a method has been proposed to inversely determine the contact forces between an installation and a building element. The 'installation' is a vibrating platform used for fitness purposes generating low frequency vertical vibrations. The used transfer functions were measured based on structural-acoustic reciprocity, for which a powerful low-frequency volume sound source was used. The resulting determined forces showed to be accurate if certain conditions are met. In the presented work, the identified forces are multiplied with simultaneously measured velocities to yield an estimate of the injected structure-borne sound power. This quantity is used as input parameter for the standard EN 12354-5 to estimate the acoustic performance of a building based on the performance of building elements, but in that standard it is not explained how to determine this parameter. The estimated injected structure-borne sound power is compared to the power found by other techniques.

1 Introduction

In many noise problems where structure-borne sound is involved, the knowledge of the contact forces between an installation and a building element is very useful. A multiplication of force with velocity at the contact points yields the structure-borne sound power, which is the quantity governing the structure-borne sound transmission, as is airborne sound power with airborne sound transmission. However, measuring the contact forces directly is a very cumbersome task. Even when it is possible to insert force sensors into the structure-borne sound path, the path might be altered in such a way that the injection of structure-borne sound power has changed as well.

2 Theoretical principles

2.1 Inverse measurement of forces

In previous work, an indirect method has been used to determine the contact forces at the contact points of an installation in an inverse way [1]. This means another active quantity (a response) than the force will have to be measured, as well as a FRF function which links this other quantity with the force. When this other quantity is the velocity, the other active quantity in mechanical problems, the method can be described to be mechanical. When the other quantity is sound pressure, as in this work, the method is called to be structural-acoustical.

For an installation with multiple contact points C_i and multiple degrees of freedom, this can be written as follows:

$$\mathbf{F}_C = \text{Sign } \mathbf{H}^{-1}\mathbf{p}_X \tag{1}$$

The FRF matrix **H** contains the FRF functions that link force and pressure between the different contact points and the different degrees of freedom. In the work done, these FRF functions were measured reciprocally as a/Q, a being an acceleration and Q a volume acceleration. In that case the value Sign equals -1. Only transversal components were considered, reducing the size of **H** to (n,n) with n the number of contact points. For 3 contact points a, b and c, 9 FRF functions have to be measured. In Figure 1 the room beneath the installation has been chosen for the positions X_i. The reciprocal way has some advantages compared to the direct way of FRF functions in the style p/F, in which case the value Sign equals +1. One of these advantages is that 3 FRF functions can be measured at once, as shown in Figure 1 at the right. Earlier measurements show a very good correspondence between measurements of a/Q and p/F, validating the use of reciprocity [1].

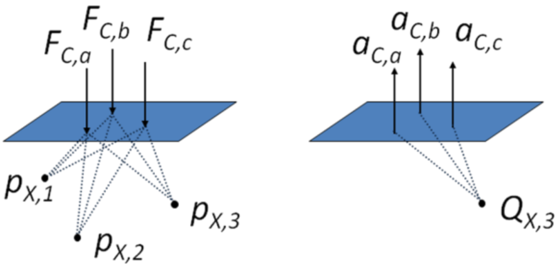

Figure 1: Measurement of pressures in the room under an operating installation on a floor (left) and determination of transfer functions $a_{C,i}/Q_{X,3}$ when the installation is removed from the building element (right)

If one isn't interested in the forces in the different contact points C_i, but rather in a single value of the total force, formula (1) can also be written as:

$$F_{C,Tot} = -<\left(\frac{a_C}{Q_X}\right)^{-1}>_{(a,b,c)} \; p_X = -<H^{-1}>_{(a,b,c)} \; p_X \tag{2}$$

2.2 Calculation of injected structure-borne sound power

When these forces are now multiplied with the velocities in each contact point, the complex structure-borne sound power is known. When the real part of this power is taken, the injected structure-borne sound power can been determined as:

$$W_{inj} = \frac{1}{2}\mathbf{F}_C^H \mathbf{v} \tag{3}$$

The superscript ^{H} denotes the Hermitian transposed, which is the conjugate transposed.

2.3 Calculation of resulting sound pressure in a room

The injected structure-borne sound power is the input parameter for the calculation of the sound pressure in any room of the building due to a structure-borne excitation of one of the building's elements, according to EN 12354-5 [2]. This standard states the acoustic behaviour of a building is determined by the properties of its elements, but remains unclear on how to determine the input parameter of the injected structure-borne sound power. It does however a suggestion for light-weight sources on heavy-weight building elements and refers to a reception plate method for this special case, outlined in EN 15657-1 [3].

3 Methods to improve matrix inversion

Since an inversion has to be applied on the FRF matrix $\mathbf{H}$, errors in the FRF functions and in the response can be magnified and seriously corrupt the solution if $\mathbf{H}$ is badly conditioned. This is mostly a problem at frequencies where the responses in the contact points C_i and/or positions X_i are governed by a limited number of eigenmodes.

A first solution to this problem is to over-determine the FRF matrix and to take more positions X_i than strictly necessary. In that case the inverse $\mathbf{H}^{-1}$ is replaced by $\mathbf{H}^{\dagger}$, which is the pseudo-inverse, calculated as $(\mathbf{H}^{H}\mathbf{H})^{-1}\mathbf{H}^{H}$. A lot of research has been conducted on which positions X_i are optimal positions, of which an example can be found in [4]. However, over-determining at low frequencies, for instance at the first eigenmode, will not improve the condition number as was found in the previous work [1] and earlier by other researchers [5] [6].

A further step that can be taken is to calculate the single value decomposition of $\mathbf{H}$ as $\mathbf{U\Sigma V}^{H}$. When the smallest singular values in the (pseudo-)diagonal matrix $\mathbf{\Sigma}$ are ignored (set to zero) and $\mathbf{H}$ is recalculated, measurement errors in the FRF matrix can be eliminated. However, one has to do this with care since also useful information can be disposed of involuntarily. Therefore, some criterions have to be set up to determine which singular values to eliminate. The simplest criterion is to eliminate only the smallest singular value at each frequency. Also an absolute value can be taken under which singular values are eliminated or a value relative to the highest singular number, e.g. 10 % of the largest singular number [7]. Finally, methods exist that eliminate singular values based on the error in the FRF matrix [8], errors in the responses [9] and errors in both [10].

In total, 10 methods were used to improve the matrix inversion. In this paper, only the best methods were selected. They are summed up in Table 1.

Method	Label
None	N
Over-determining with 5 positions X_i for the pressure instead of 3	O
Elimination of the smallest singular value	S1
Elimination of singular values smaller than 10 % of the highest singular value	S3
Elimination of singular values of which the sum of the concerned value and all smaller singular values is smaller than a measure of the errors in the response [10]	S9

Table 1: Used methods to avoid matrix inversion problems and corresponding labels as used in the following figures.

4 Experimental setup

4.1 Vibrating source and floor

The installation under which the forces have been measured in the inverse way, is a vibration plate used for fitness purposes and physical therapy of the company Fitvibe. It can generate vertical vibrations with amplitudes up to 3 mm according to the technical information. The frequency ranges between 20 Hz and 60 Hz, adjustable in steps of 1 Hz. The vibrations are created by two motors with eccentric masses on their axles that rotate in phase but in different senses, producing a dominant vertical vibration. Recorded acceleration spectra show not only a sine at the rotation frequency, but also higher harmonics due to minor imperfections in the motor mechanism like rub between rotor and stator, rub inside ball bearings and mechanical looseness in general [11].

The floor on which the vibrating plate has been placed, is a concrete floor resiliently mounted in an opening between two transmission rooms at the laboratory of the K.U.Leuven. Figure 2 shows a picture of the vibration platform on the concrete floor in the upper transmission room and a construction scheme showing the 3 contact points. It must be noted that the relation between the size of the opening (2 x 2 m²) and the volume of the transmission rooms (87 m³), is not the same as in real buildings, where the floors are generally (slightly) larger and the room volumes (slightly) smaller.

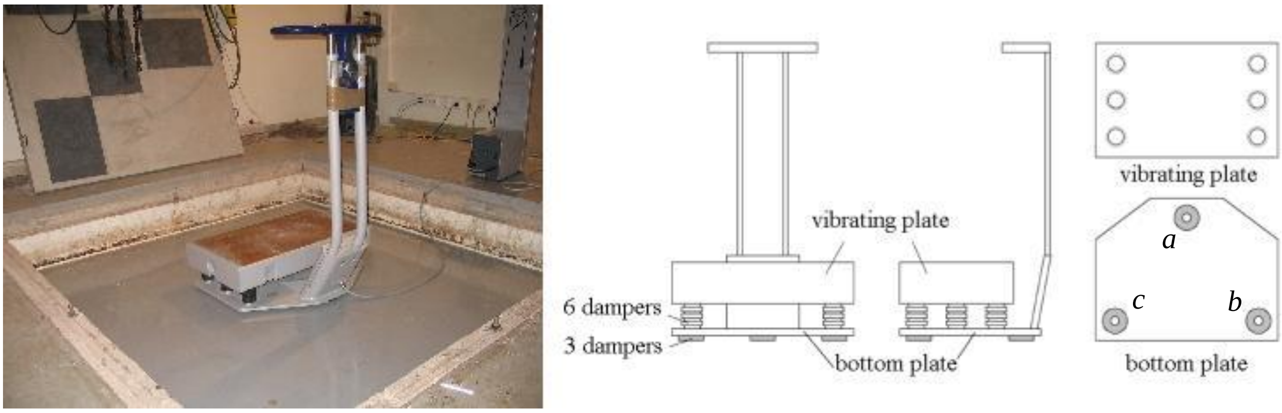

Figure 2: Vibration plate on concrete floor and construction scheme with labeled contact points.

As operating conditions frequencies of 24 Hz, 40 Hz and 60 Hz were chosen. Because of the strong tonal character of these operating conditions, a 4th broadband "operating condition" was created by manually tapping the bottom plate once at a certain point with an impact hammer (see Figure 3). The registered force in the built in force sensor of the impact hammer was used to normalize the resulting active quantities. Because the resulting signals are transient and not stationary, the spectral density had to be calculated instead of a power spectrum.

Figure 3: Location of impact of the broadband impulse excitation of the bottom plate of the vibrating plate by use of an impact hammer. The location is situated at the tip of the hammer.

4.2 Exciter of volume acceleration

The room in which the positions X_i are taken, is the lower room of the two. The exciter generating the volume acceleration Q is a prototype of a powerful low-frequency volume point source of the company Qsources (Figure 4). The point source behaviour is a necessary condition for equation (1) to be used. The powerful low-frequency behaviour is necessary to measure inverse forces in the low-frequency range as with the vibration plate. The source has been equipped with a sensor that measures a signal proportional to the volume acceleration. The sensitivity of the sensor has been determined in a separate measurement in semi-free field conditions.

Figure 4: Volume point source in lower transmission room.

4.3 Direct measurement of the force and the velocity

The force was also measured directly during the measurements to compare the inversely measured force with. Force sensors were inserted between each contact point. Because of the large circular shape of the contact points of the vibration plate, each force sensor was squeezed between two aluminium cylinders with a diameter comparable with these of the contact points (Figure 5). The thickness of the cylinders has been chosen to be 2 cm to be sure the resulting “sandwich” has its first resonance frequency well above the frequency range of interest.

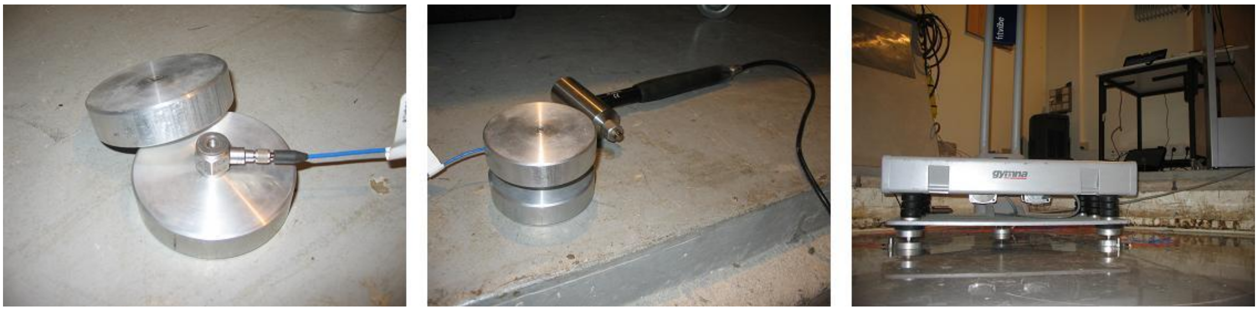

Figure 5: Mounting of the force sensors between two aluminium cylinders and arrangement under the vibration plate.

The velocity was measured simultaneously as closely as possible next to each contact point in order to find the injected structure-borne sound power later on.

5 Comparison of directly and inversely measured forces

In Figure 6, the directly and inversely measured force at contact point *a* are shown for the tonal operating condition of 24 Hz. For the inversely measured force, the method S3 was used to improve the inversion. As is visible from the figure, matrix improvement according to method S3 is mostly efficient in the frequency range of 25 to 60 Hz. This corresponds with the first eigenmode of the system floor-room as was concluded in earlier work by showing a plot of the condition number of the FRF matrix [1]. Analysis showed that in this frequency range, only 1 singular value was maintained as only 1 singular value is capable to describe the system completely within the neighbourhood of the first eigenmode. Therefore it is best to only maintain 1 singular value in the vicinity of the first eigenmode of a system, which can be easily found by plotting a FRF function, the condition number of a complete FRF matrix or the mobility.

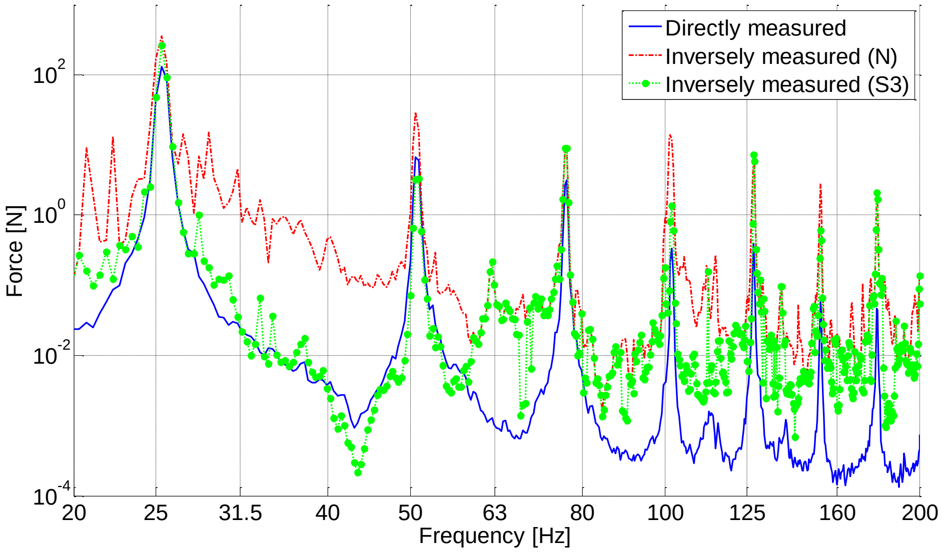

Figure 6: Comparison of the directly measured force and the inversely measured force at contact point *a* with a tonal operating condition of 24 Hz of the vibrating plate on the concrete floor (spectrally). Also the inversely measured force with improvement of the inversion through method S3 is plotted.

When the same is plot after weighting the results in third-octave bands, one gets the result as in Figure 7. It appears that at higher frequencies, the method S3 also delivers good work because the inversely measured force using method S3 better fits the directly measured force than the inversely measured force without any effort of improving the inversion. From 800 Hz onwards, the volume point source didn't behave linearly anymore, which explains the bad correspondence at 800 Hz. The conclusion is that improving the inversion seems a logical and necessary step.

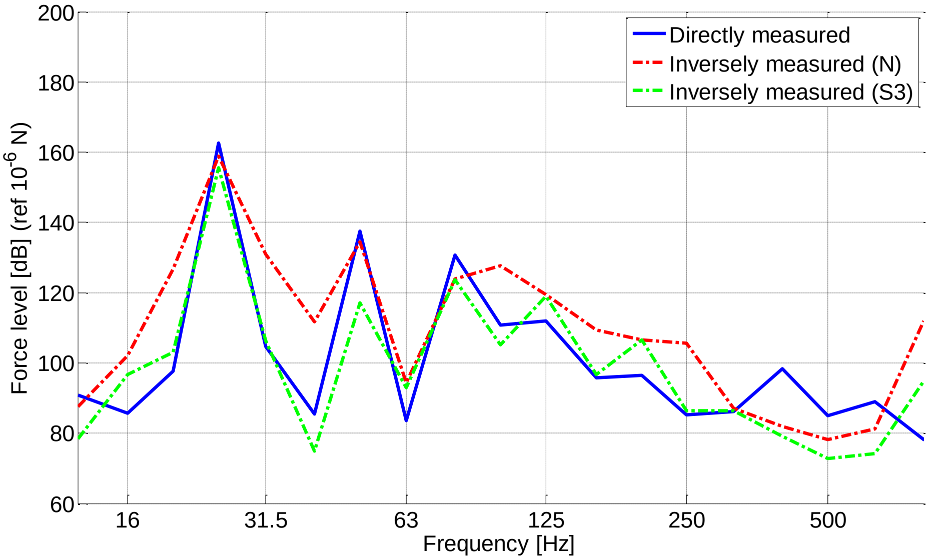

Figure 7: Comparison of the directly measured force level and the inversely measured force level at contact point *a* with a tonal operating condition of *24 Hz* of the vibrating plate on the concrete floor (in third-octave bands). Also the inversely measured force with improvement of the inversion through method S3 is plotted.

Figure 8 shows the same as in Figure 7 but for a tonal operating condition of 40 Hz of the vibrating plate. The importance of improving the FRF matrix inversion is accentuated as the excitation peak at 40 Hz lies closely to the first eigenmode of the system and the error between directly measured force and the inversely measured force without any improvement is clearly visible. A resonance phenomenon was indeed physically observable in the sound pressure in both rooms and in the velocity spectrum of the floor. However, when the method S3 is applied at the matrix inversion, the resulting inversely measured force seems to underestimate the directly measured force at the resonance frequency with 10 dB.

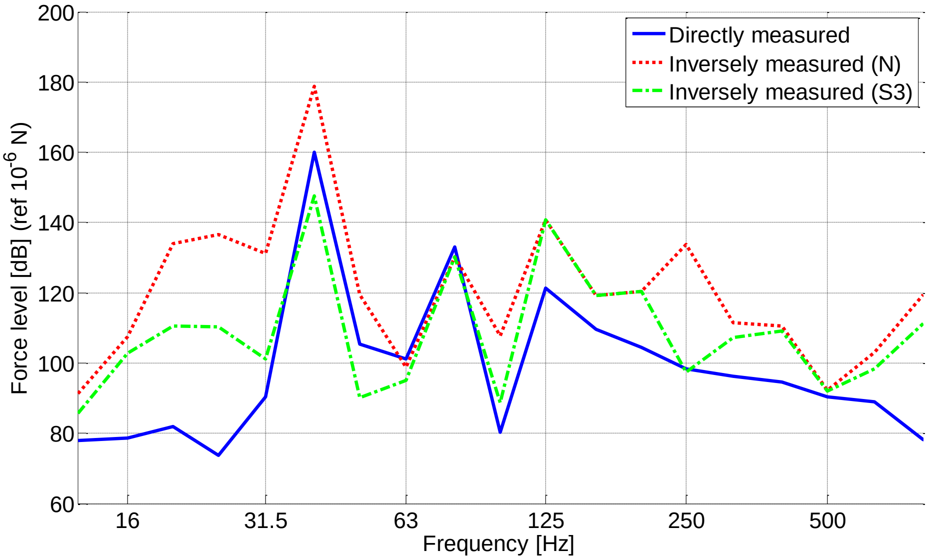

Figure 8: Comparison of the directly measured force and the inversely measured force at contact point *a* with a tonal operating condition of *40 Hz* of the vibrating plate on the concrete floor (in third-octave bands). Also the inversely measured force with improvement of the inversion through method S3 is plotted.

Also at higher frequencies the correspondence is not that well, but there, the inversely measured force with method S3 is *over*estimating the directly measured force. As was suspected in earlier work, this might be due to an airborne sound transmission between the bottom plate and the floor that is relatively stronger than structure-borne sound transmission. Indeed will this large, hard and flat bottom plate radiate a lot of sound to nearby surfaces when in strong resonance. Therefore, one has to pay careful attention at possible important airborne sound transmission, which is not captivated in the (structure-borne) model.

When the broadband operating condition with the impact on the bottom plate is considered and the forces are again compared at contact point *a* as in Figure 9, the necessity of improving the matrix inversion again seems obviously. In contrary to the tonal operating conditions, there was the ability to use over-determining (method O) in the broadband operating condition. In this figure, also the methods S1 and S9 are shown. It appears that over-determining the system is really useful at all frequencies except in the vicinity of the first eigenmode. This corresponds to earlier conclusions and the conclusions of Thite [1] [5] [6]. Improving the inversion by eliminating singular values further optimizes the correspondence between directly and inversely measured force. Only eliminating 1 singular value (and keeping 2) is not sufficient at the first eigenmode (method S1), as explained earlier in this paper. Method S3 doesn't seem so fruitful in this case. Attaching the singular value eliminating criterion to errors in the response delivers good work in this case (method S9).

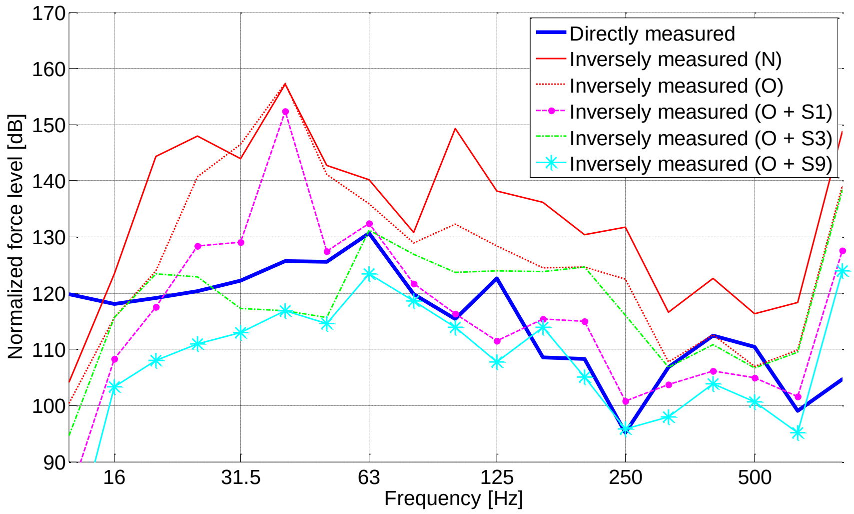

Figure 9: Comparison of the directly measured force spectral density level and the inversely measured force spectral density level at contact point *a* with a *broadband* operating condition of an impact on the bottom plate of the vibrating plate on the concrete floor (in third-octave bands). Also the inversely measured force with improvement of the inversion through method S3 is plotted.

Figure 10 shows the total force, summed over the 3 contact points. In some cases, knowledge of the detailed distribution of the force isn't required after all. Another reason to use a total force might be that the contact points are too close to each other and the forces are expected to be of an equal nature anyway. Also line contacts or surface contacts can lead to using such an approach. Both the matrix formulation (1) as well as the scalar formulation (2) are used to inversely determine the measured force. As expected, the scalar formulations, that look at the vibrating plate as if having only 1 contact point, work very well. Over-determining barely improves the result. The matrix formulation involving the inversion doesn't perform well if the inversion isn't improved in any way. Over-determining already improves the result, as in the case of the force in contact point *a* in Figure 9. A result resembling the success of the scalar formulation is attained when singular value elimination according to method S1 is applied.

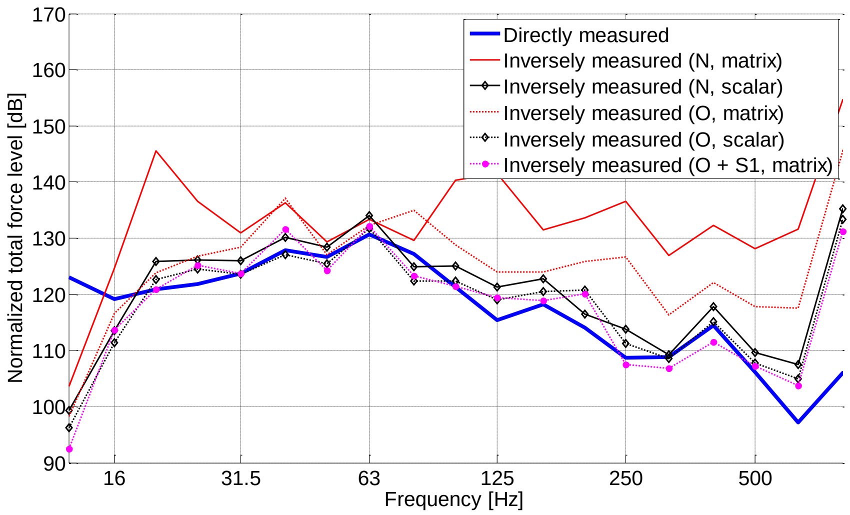

Figure 10: Comparison of the directly measured *total* force spectral density level and the inversely measured *total* force spectral density level with a broadband operating condition of an impact on the bottom plate of the vibrating plate on the concrete floor (in third-octave bands). Both the matrix formulation and the scalar formulation were used to determine the inversely measured force, as well as some improvement methods concerning the inversion in the case of the matrix formulation.

6 Comparison of injected structure-borne sound power

Since the velocity has been measured simultaneously with the pressure, the injected structure-borne sound power was calculated according to formula (3), both with directly as with inversely measured forces. Also the reception plate method has been applied to calculate the structure-borne sound power, with the modification that the floor itself has been used as reception plate (called "artificial RPM"). This is a feasible assumption since the plate is resiliently mounted in the opening between the transmission rooms, forcing injected power to be completely dissipated in the vibration since no edge losses exist. The injected power can be calculated by expressing the balance between injected power and dissipated power, determined by the loss factor and the surface-averaged squared velocity field on the plate as in EN 15657-1 [3]. The reception plate method is a scalar method as no distinction is made between different contact points.

The measured space-averaged sound pressure level in the lower transmission room is compared with the predicted sound pressure level according to EN 12354-5 with the above-mentioned injected structure-borne sound powers as input [2]. The result is shown in Figure 11 for an operating condition of 24 Hz of the vibrating plate on the concrete floor. The overestimation of the artificial reception plate method at lower frequencies is due to the fact that this was determined when the vibrating plate was not resting on the 3 force sensor sandwiches. The bottom plate was really close to the surface of the concrete floor enabling airborne sound to transfer to the floor. This effect is not present in the (inversely) measured pressure that was determined when the vibrating plate was standing on the sandwiches.

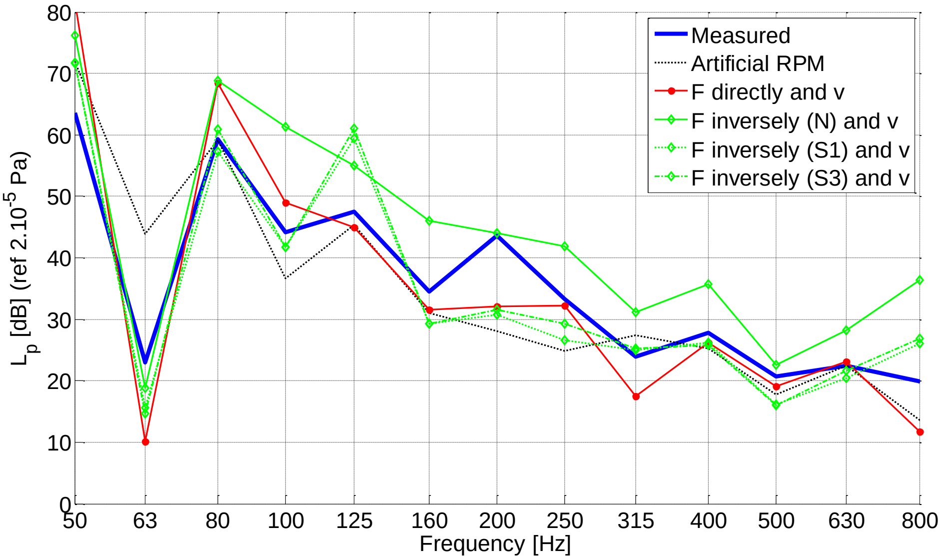

Figure 11: Comparison of the measured sound pressure level and the sound pressure level predicted via the calculation method outlined in EN 12354-5 which uses the earlier mentioned structure-borne sound powers as input parameter. The sound pressure level in the lower transmission room is concerned while the vibrating plate is in the tonal operating condition of *24 Hz* on the concrete floor.

The predicted pressures calculated with the force and velocity as input parameter are very similar except the one where the force is determined inversely without any application of improvement at the inversion stage. This again demonstrates the usefulness of improvement techniques. However, still differences up to 10 dB with the measured pressures are noticeable. It must be mentioned that the calculation procedure in EN 12354-5 involves a lot of parameters and assumptions that cannot be detailed in this paper, but that can cause certain deviations between measured and predicted quantities. In any case, the prediction is difficult in the lower frequencies because the calculation procedures are based on statistical energy analysis, which supposes an infinite eigenmode density in all frequency bands. This is obviously not the case in the lower frequency bands, since the lowest eigenmode of the system floor-room is situated around 40 Hz.

In Figure 12, the same is plotted as in Figure 11, but then for a tonal operating condition of 40 Hz. Again, the artificial reception plate method yields different results in the lower frequency bands because of the slightly different setup of the vibrating plate. The similarity between the results with the directly measured and the inversely measured forces is however lost, especially for frequencies higher than 125 Hz. This is due to the earlier mentioned dominating airborne sound transfer between the bottom plate and the floor. This effect causes the floor to vibrate more than only through structure-borne sound excitation. The vibrating floor radiates into the lower transmission room, where a pressure is measured that is not exclusively based on structure-borne sound transmission. When calculating the inversely measured force out of this pressure, an overestimation of the directly measured force is the result. Finally a higher sound pressure level is calculated than this using the directly measured force because of this overestimated force. However, since the measured pressure naturally contains this airborne sound transfer component, the resemblance is better for the predicted pressures that use the inversely measured force!

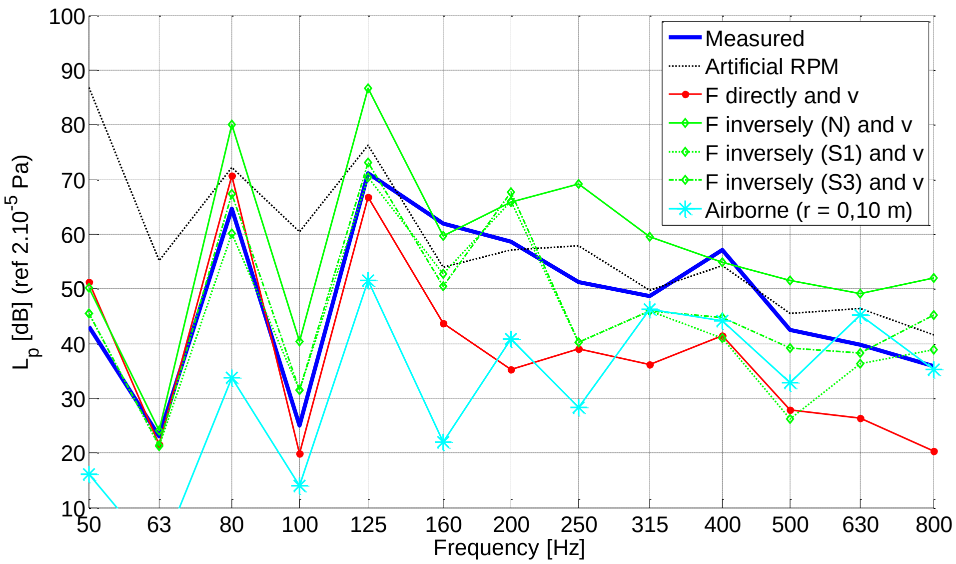

Figure 12: Comparison of the measured sound pressure level and the sound pressure level predicted via the calculation method outlined in EN 12354-5 which uses the earlier mentioned structure-borne sound powers as input parameter. The sound pressure level in the lower transmission room is concerned while the vibrating plate is in the tonal operating condition of *40 Hz* on the concrete floor. Also the predicted airborne sound level pressure is plotted for the source centre to be at a distance of 0,10 m of the floor.

Finally, in Figure 13 the same as in Figure 12 is showed for the broadband operating condition of an impact on the vibrating plate's bottom plate. In this case the artificial reception plate method was used in the same setup as the other methods. The prediction via the directly measured force is the best one. There is an overestimation of the pressure level by the methods that use the inversely measured forces, which is thought to be due to noise in the pressure. The measured pressures in the lower room were indeed much smaller with the broadband operating condition than with the tonal operating conditions.

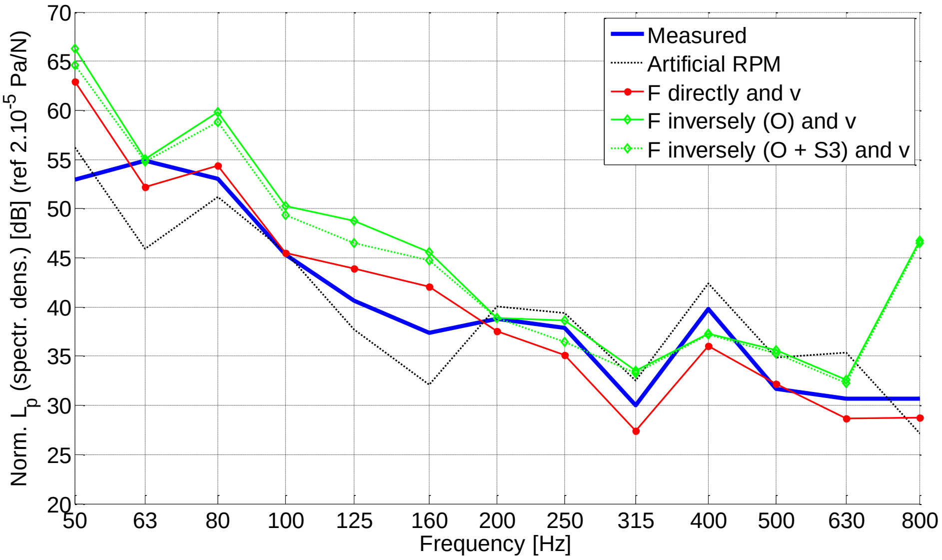

Figure 13: Comparison of the measured normalized sound pressure spectral density level and the normalized sound pressure spectral density level predicted via the calculation method outlined in EN 12354-5 which uses the earlier mentioned structure-borne normalized sound power spectral densities as input parameter. The normalized sound pressure level in the lower transmission room is concerned while the vibrating plate is on the concrete floor and in the broadband operating condition of an impact on its bottom plate.

7 Conclusions

A structural-acoustical inverse method was used to determine the contact forces at the contact points of an installation to avoid the difficult direct measurement. These forces can be used in calculating the injected structure-borne sound power, a quantity that is used as input parameter in EN 12354-5 to calculate the sound pressure level in any room of a building, if the velocity at the contact points is measured simultaneously. For the FRF matrix involved in the calculation, a structural-acoustical reciprocal method was applied with a volume point source generating a volume acceleration and causing an acceleration of the concerned building element. This volume point source is powerful and has excellent low frequency properties in order to be used to calculate the low frequency forces at the contact points of a vibrating plate used for fitness purposes. The reciprocal technique is advantageous over the direct technique when the contact points are brittle or difficult to reach and because several transfer functions can be measured at once.

Because a matrix inversion is involved, several methods are used to improve the inversion. These are based on over-determining the system on one hand and on eliminating singular values of the FRF matrix on the other hand. It must be noted that this experiment on a compact piece of floor in a more or less rigid boundary is a worst case for the inversion. When looking at the installation as having only 1 contact point, a scalar formulation can very accurately determine the total force, which is often satisfying at lower frequencies.

Comparison of the inversely measured forces with directly measured forces revealed several issues that have to be treated with vigilance. First, other transfers of sound power are possible than those represented by the FRF functions in the FRF matrix. They can of a structure-borne kind through a degree of freedom not comprised in the model, or they can be of an airborne kind. The latter can be mostly problematic when the source has a large, flat plate close to the surface of the building element that radiates and transfers a lot of airborne sound energy. These other transfers of sound power will yield an overestimation of the directly measured force.

Second, the pressure measured to calculated the inversely measured forces must not be contaminated with noise, since this also yields an overestimation of the directly measured force.

Third, improvement methods are required for the inversion of the FRF matrix. Over-determining is a very good first step but cannot improve the situation at the first eigenmode of the system. Eliminating all singular values except one is a necessary tool in the vicinity of the first eigenmode. Eliminating the smallest singular value or eliminating all singular values smaller than a certain percentage of the highest singular value are also good methods. When the response is rather noisy, eliminating singular values based on errors in the response can bring help.

Comparison of the measured pressures in the room underneath the installation with the predicted pressures using directly and inversely measured forces, shows that the disadvantage of the risk using the inverse method when also airborne sound transfer is involved, is now transformed into an advantage! This is because the predictions with directly measured forces will underestimate the measured sound pressure that includes the airborne sound transfer component.

The conclusion is that the structural-acoustic inverse measurement of the force is **beneficial compared to the direct measurement** of the force when using this force in the calculation of a sound pressure prediction. However, one has to be aware that the inversely measured forces might not be the actual forces. Rather they are some kind of **pseudo-forces** in the case of other important power transfers than only transversal structure-borne ones. The result is a better prediction of the pressures when they are multiplied with the measured velocities than by using directly measured forces, if even available.

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